Economic Growth and Finance.
A cointegration analysis in US and Japan

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A cointegration analysis in US and Japan

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ABSTRACT: This paper aims at investigating the relationship between financial and economic development for two countries: the US and Japan.

A great deal of theoretical and empirical studies showed the existence of a close relation between the development of the financial sector and economic growth (Greenwood and Jovanovic, 1990, Bencivenga and Smith, 1991, King and Levine, 1993, Levine et al., 2000); nevertheless many concerns still remain: it is, for instance, unclear how the development of financial markets drives economic growth and, more relevant, whether it causes or is caused by economic growth. Moreover, previous empirical studies showed that time series and cross sectional approaches lead to different results.

In this paper, the long-run relationship among finance and growth is investigated through the cointegration analysis (an estimation method developed over the last decade). The cointegration analysis can help to shed light on the aforementioned issues: it helps both to examine the interactions between the variables under consideration (real GDP per capita, private credit, investment share and inflation), taking into account the non stationarity of the data, and to capture the existence of potential cointegrating links between series (being explicit a priori about their form).

With this regard, the aim of our analysis is twofold: 1) to investigate whether it is possible to find a stable relationship between financial development and real GDP per capita; 2) to investigate the possible channels of transmission from financial intermediation sector to economic growth.

JEL classification: O16, E44, C22
Keywords: Economic growth, finance, cointegration analysis

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References
1. Introduction

‘Why do some countries grow rich while others remain poor? Do countries converge to steady state paths? Are countries poor because they lack inputs or because of technological and financial differences? What happens in the long run?’

The study of economic growth is inspired by these research questions. Theoretical works of the late ‘80s (William Baumol, 1986; Robert Lucas, 1988; Paul Romer, 1986) suggest that growth is mainly driven by technical advances: growth differences are due to the differences in technology.

Along the line of research of Lucas (1988) and Romer (1986), Alderman et al. (1996) identify in human capital and in research and development (R&D) new variables in explaining growth patterns. Specifically, the authors support the view that poor countries grow faster than developed countries since ‘relatively’ poor economies have lower stocks of human and physical capital; consequently, the marginal product of extra capital of these economies is high and their growth is faster.

In recent years, researchers have been interested in the identification of the new variables able to explain growth process. Some of them have looked at the potential effects of financial development on growth.

The well-known works of King and Levine (1993), for instance, shed light on this issue, suggesting a positive relationship between the financial system and economic growth. The main argument, supported by these authors, is that the good performance (‘functions’) of financial intermediaries affects growth. The functions of the financial system to which King and Levine (1993) and Pagano (1993) refer are: resource pooling, resource transfer, liquidity risk amelioration and transaction cost reductions. According to them, these functions form channels through which finance drives economic growth.

Regarding the identification of the possible channels of transmission, there has also been intense research which has produced diffuse theoretical and empirical works. More precisely, the main question of these works is to discover how to model the connections between the functions of the financial system and economic growth.

This work takes this matter seriously. Furthermore, it relates closely to two papers that attempted to explain finance-growth relationship using cointegration analysis: Demetriades and Hussein (1996) and Arestis and Demetriades (1997)’s works.
Demetriades and Hussein (1996) use macro data to conduct the cointegration analysis. Specifically, they test the existence of a stable relationship between at least one of the financial development indicators (private credit, liquid liabilities) and real GDP per capita. Nevertheless, their focus is to test the causality between financial development and real GDP, whereas our main focus is to identify the channels of transmission from the financial sector to real GDP per capita.

Arestis and Demetriades (1997) also use macro-data to test the influence of financial development on growth. By running a Johansen cointegration analysis for two countries (the US and Germany), they find the existence of two cointegrating relationships. In Germany, the first cointegrating vector shows a positive relationship between real GDP per capita and banking system development and a negative relationship between stock market volatility and real GDP. The second cointegrating vector shows a positive relationship between the stock market capitalisation and banking system development. In the US, the interpretation of the two cointegration vectors is more complex, because of the endogeneity of stock market capitalisation. In substance, Arestis and Demetriades (1997)’s paper differs from our analysis: they empirically assess the possibility that stock markets might affect growth through liquidity (see, among others, Levine and Zervos (1996)’s work), whereas we aim at testing the hypothesis that the financial system may affect real GDP through the investment share.

This work is made up of 6 sections. Section 2 frames the theoretical context: it presents a theoretical review of the literature on economic growth which has focused on the identification of the channels of transmission from financial intermediation to economic growth. Two simple models are presented. The first model is the theoretical framework developed by De Gregorio and Guidotti (1993), in spirit of the works of Goldsmith (1969), McKinnon (1973) and Shaw (1973). It investigates how the investment share, which is related to the abilities of financial intermediaries, affects real GDP per capita. In this framework, the financial intermediaries interact with savings and with the marginal productivity of capital. The second model presents the framework developed by Fischer (1993), De Gregorio (1993) and Boyd et al. (2001), investigating how inflation, affecting the ability of the financial intermediaries in identifying better investment opportunities, influences real GDP per capita.
Sections 4 and 5 provide evidence on the macro-channels of transmission identified in Section 2. Here, it is presented a comparative analysis between two countries -representative of two different financial, economic and social backgrounds (the US and Japan) - testing for the existence of cointegrating relationships. The aim of this analysis is to investigate whether real GDP per capita is affected by exogenous forces like financial development. Finally, Section 6 concludes.

2. Financial intermediation and economic growth: a theoretical overview

The nexus linking financial development and economic growth has been thoroughly investigated since the pioneering contributions of Goldsmith (1969), McKinnon (1973) and Shaw (1973). The overall (financial and economic) literature documents the importance of financial development for the economic growth.

However, in recent years, there have been several studies which attempted to analyse different aspects of this relationship. For instance, a strand of literature has investigated whether financial development leads to good economic performance and, contextually, it has analysed the degree of strength of this relationship. Another strand, instead, focused on the identification of the channels of transmission from financial intermediation to economic growth.

Since we are interested in testing growth theory through the cointegration analysis, it seems worthwhile looking at diverse opinions on the possible channels of transmission which the literature has formulated.

The following sections look at two theoretical models\(^1\): the first model is concerned with the impact of the investment share on economic growth, while the second model deals with the impact of the inflation on financial intermediaries’ performance.

\(^1\) For a detailed review of the literature, see Testa (2005).
2.1 On the role of investment on economic growth

To begin with, let us assume² a production function depending on capital stock:

\[ y_t = f(k_t) \quad (1) \]

where \( y_t \) and \( k_t \) denote output and the capital stock at time \( t \), respectively.

By differentiating the equation (1) and denoting the rate of growth of output by \( \hat{y} \), the saving rate \( \left( \frac{dk}{y} \right) \) by \( s \) and the marginal productivity of capital by \( \phi \), we have:

\[ \hat{y}_t = \frac{dk_t}{y_t} f'(k_t) = s \phi_t \quad (2) \]

By equation (2), the rate of output growth is the product of savings rate and the marginal productivity of capital. In terms of the equation (2), two possible routes of influence of financial development on economic growth can be found. First, the development of financial markets may enhance the efficiency of capital accumulation (increasing \( \phi \)). Second, financial intermediaries can contribute to increase the saving rates and, thereby, the investment rate (increasing \( s \)).

Goldsmith (1969) was among the first to argue that the financial intermediaries affect economic growth, since they contribute to more efficient allocation of capital stock.

‘The financial superstructure accelerates growth and improves economic performance to the extent that it facilitates the migration of funds to the best user, i.e. to the place in the economic system where the funds will yield the highest social return’. (p.400)

‘The effect of financial institution on economic growth must therefore be investigated from the point of view of both the total volume and the allocation of saving and investment’ (p.395)

McKinnon (1973) and Show (1973) offer another explanation of the systematic relation between per capita output and financial development. They support the view that financial depth imply not only higher productivity of capital but also higher saving rates and, hence, a higher volume of investment.

In the light of the effects of the financial repressions on savings and investment occurred over the ‘70s, McKinnon and Shaw support that these policies, which resulted
in negative real interest rates, reduced the incentives to save. Hence, lower savings are associated with lower investment and growth.

Theoretical works of Greenwood and Jovanovic (1990) and Bencivenga and Smith (1991) represent two alternative visions of the role played by the financial system on the economic growth.

More precisely, Greenwood and Jovanovic develop a macroeconomic model of growth in which financial intermediaries affect economic growth, by lowering information costs and by sifting capital towards more efficient investment opportunities. In their model, higher growth rates are associated with higher investment rates, which are, in turn, caused by any factors leading to a better allocation of economic resources. These factors capture essentially financial intermediaries’ functions. Put in other words, the importance of finance supply, provided by financial intermediation sector, comes from the consideration that intermediate finance encourages efficiency by providing a discipline for consumers and firms’ managements and facilitating the transfer of savings into investments.

The model, as set out here, assumes that all consumers maximise their expected utility over time: that is, the expected flow of consumption is maximised after having chosen the amount of capital to be saved. More specifically, this model assumes that, once the amount of saving has been decided, individuals can choose how much of saved capital to invest in safe assets but with low return and/or how much to invest in risky assets but with high return. This assets’ combination will be chosen by consumers within financial intermediation sector and it will take into account the presence of aggregate and idiosyncratic risk.

Greenwood and Jovanovic solve the above optimisation problem by using the dynamic programming approach. This approach analyses the decisions made by individuals across periods; in particular, it compares two situations: the situation in which individuals decide to invest by themselves with the situation in which individuals decide to be advised by intermediation sector. The solution of the two value functions deliver a policy function for each function, which determines the optimal saving for the level of capital. The result of the value function for individuals, who delegate the

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2 The formulation of this comprehensible economic model is due to De Gregorio and Guidotti (1995).
financial intermediaries the management of their savings, is $s(k_t) = \beta k_t$, i.e. the agents within the intermediation sector would save a constant fraction of their wealth each period; whereas the policy function for individuals who prefer to operate outside the financial intermediation sector is $s(k_t) > \beta k_t$. It means that the agents outside the intermediation sector will save an amount of money which is greater than the amount saved by individuals who perform inside it; on the contrary, on the side of returns, the agents who live under autarky receive lower returns on their investment then those who operate within the financial intermediation sector. Placing this analysis within the framework of income distribution and considering that for very poor agents the saving rate is exceptionally low, the conclusion that Greenwood and Jovanovic draw is that, since both groups have the same saving rate and since members of intermediation sector face a better distribution of returns of their investments than poor agents, the wealth of agents inside the financial intermediation sector is greater than the wealth of agents who are self-financed.

Bencivenga and Smith (1991) present an endogenous growth model with multiple assets. Individuals choose to invest in liquid assets and/or illiquid assets. The main task of the financial intermediaries is to help individuals to reduce the risk associated with their liquidity needs. In fact, in the absence of financial intermediaries, individuals might be obliged to convert their investment in liquidity.

In the Bencivenga and Smith’s model, the objective of individuals is to maximise a utility function with argument the consumption of the second and third generation. It is assumed that households save fixed proportions of their income and invest them in liquid and illiquid investments, through bank intermediation. In particular, it is argued that households, who liquidate their investment prematurely, receive lower rate of returns than those who wait for two periods. Optimisation takes place at two different moments: under autarchy and under the presence of bank intermediation. In both cases, three equations define growth rate of equilibrium in this model of economic growth: market clearing condition, the individuals’ optimisation and the labour market clearing condition.

The key predictions of the economic growth model with the presence of financial intermediaries are:
(1) A higher equilibrium growth rate is associated with a greater fraction of savings ($\Phi$) invested in the accumulation of productive capital, holding fixed $\theta$ (labour’s share), $R$ and $\psi$ (rates of return).

(2) A higher equilibrium growth rate of capital is associated with higher values of rate of return $R$, with $\Phi$ (fraction of savings) held fixed.

(3) A higher labour’s share in output ($1-\theta$) is associated with a higher equilibrium growth rate of capital, keeping fixed $\Phi$ (fraction of savings).

Along the same line, De Gregorio and Guidotti (1993) analyze the effect of financial development on growth through the channel of the investment. In their evidence, they find that the negative relationship between financial intermediation and growth come from a negative effect on the efficiency of investment, rather than on the volume of investment.

Recent theoretical and empirical assessments of the financial determinants of economic growth include the studies by Rajan and Zingales (1998), Carlin and Mayer (2003), and Guiso et al. (2004). At different level -micro level- these recent studies provide further evidence of the importance of the financial system’s structure for the economic growth.

Rajan and Zingales develop a model of industry growth in which the impact of external finance on industries’ growth is related to the fact that their capital structure and financial capacity is determined by the degree of financial development of a country.

The relevant issue, that the authors deal with, is to analyse the extent to which diversity in the ways industries finance their investment, consistent with the degree of financial development of country, cause the variation in industries’ growth. In this context, developments in the financial sector of a country influence the source of funding of an industry (especially, by lowering the external finance) and, in turn, affect its production technology.

To investigate this issue, they apply a panel cross-country regression to assess whether, in the environment of the industry, the sources of investment finance and the degree of financial development of a country are important for industries’ economic growth.

An empirical evidence of positive effect of financial integration on the economic growth of firms can be also found in the recent study of Guiso, Jappelli et al. (2004). In
Italy, Guiso, Jappelli et al. look at the role of the financial market in financing small and medium enterprises. They suggest that the way in which financial integration takes place has an important effect on SMEs and on large firms. They conjecture that “if it occurs via massive entry of highly efficient intermediaries in previously protected markets dominated by inefficient local banks, small businesses will benefit a lot. If instead integration will occur by giving firms access to other countries’ securities markets, then only large businesses will benefit”. The differences in the nature and effectiveness of financial integration will affect the source of investment finance of Italian firms and, then, their economic growth.

2.2 On the role of inflation on economic growth

The potential links between inflation and growth have been discussed and developed in Fischer (1983), Fischer (1991), De Gregorio (1993), Barro (1995) and in Boyd et al. (2001) among others. The negative effect of inflation on growth suggests that higher rate of inflation reduces the level of investment and the productivity of capital and, by extension, through the new growth theory mechanism, this interaction would produce a negative relation between inflation and growth.

In Fischer (1983, 1991), the inflation rate is defined as the best indicator of the overall ability of the government to manage the economy. He argues that the main reason of expecting a negative relationship between inflation and growth is that “a government that is producing high inflation is a government that has lost control. Economic growth is likely to be low in such economy”. He also extends his analysis to the variability of inflation which represents a more direct indicator of the uncertainty of the macroeconomic environment.

Fischer’s findings suggest that one important route through which inflation affects economic growth is by reducing investment and by reducing the rate of productivity of capital.

Unlike Fischer (1983, 1991), De Gregorio (1993) develop two theoretical models, identifying different channels through which inflation affects growth. The first model focuses on the effects of inflation on the investment rate. The second model studies the effects of inflation on the productivity of capital (measured by the ratio of GDP per capita growth to investment rate). His empirical evidence suggests that the
level of inflation and its variability has negative effects on economic growth and that the main channel through which inflation affects growth is through the reduction of the productivity of capital.

Differently, Barro (1995) uses the neoclassical growth framework developed by Barro and Sala-i-Martin (1994) to assess the effect of inflation on economic growth. The general notion that he adopts in this framework is that the government policies and private-sector choices determine where the economy would go in the long run. He argues that, if a poor country selects unfavourable policies, then its growth rate will not be high and it will not tend to catch up richer countries. His major funding is that the effects of inflation on growth and investment are significantly negative.

In 2001, Boyd et al. empirically assess the predictions that increases in the rate of inflation interfere with the ability of the financial sector to allocate resource effectively. They use a cross country regression to test the finance-inflation relationship. Their findings suggest that there is a strong negative association between inflation and (a) financial sector’s lending to private sector, (b) the quantity of bank assets and (c) the volume of liabilities issued by banks. As inflation rises, the marginal impact of inflation on banking lending activity and stock market development diminish rapidly.

3. Cointegration analysis

Although, as we saw so far, there is considerable (empirical and theoretical) literature that suggests that the existence of well-developed financial markets improves the growth rate of an economy, it is somehow surprising that empirical studies which attempted to establish the type of the relationship between financial development and economic growth in the context of cointegration are few.

In the following sections, we describe the concept of cointegration, useful for the understanding of our empirical analysis.

3.1 Investment, saving and financial development data measurement

As already mentioned, the McKinnon-Shaw hypothesis suggest that the main channel of transmission is the real interest rate which stimulates financial savings and financial intermediation’s activities, thereby increasing the supply of credit to the
private sector. This, in turn, stimulates investment and growth. This point of view has been supported by several studies. Nevertheless, there are other studies (see, among others, De Gregorio and Guidotti, 1995) which argue that real interest rates are poor indicators of the degree of financial intermediation and suggest the efficiency of investment as channel through which financial intermediation affects growth.

In other words, although the saving ratio\(^3\) (savings as a share of GDP) features in the main growth models (recall that one prediction of the models is that higher saving ratios will be associated with higher growth rates), it seems justifiable to look at domestic investment shares of GDP.

Among various indicators of financial development, we use private credit, because we believe that it is more closely related to the level and efficiency of investment and hence to economic growth.

This measure, constructed dividing the credits issued to the private sector for GDP, attempts to capture the role of financial intermediaries in channelling funds to private sector. It excludes the credit issued by the central bank and by other monetary authorities. It corresponds to lines 22d and 42d drawn from the International Monetary Fund’s *International Financial Statistics*.

3.2 Unit Roots and cointegration: long-run behaviour and econometric concepts\(^4\)

This study goes on to consider the methodological issue of the cointegration analysis. Introduced by Granger (1981) and Engle and Granger (1987), the cointegration analysis is an important econometric tool of the last decade. It captures, in statistical terms, the typical characteristics of time series. More importantly, it links the economic notion of a long run relationship between economic variables to a statistical model, since it helps to clarify the “spurious regressions” or “non-sense correlations” problem associated with trending time-series data.

\(^3\) Saving has rarely appeared as an explanatory variable in growth regressions, perhaps because it is felt that its role is better captured by investment, perhaps because of inadequate data.

\(^4\) This section on the econometric issue is drawn from Ericsson, (1992).
In order to derive the implications of the cointegration for our economic analysis, we think that it is more fruitful to introduce before the basic concepts and then the econometric concepts.

To make things easier, let us begin by considering our system of four variables \((LY, LINV, LINF, LPC)\), where \(LY\) is the log of real GDP per capita, \(LINV\) is the log of investment share, \(LINF\) is the log of inflation and \(LPC\) is the log of private credit. These four variables may be linked together by behavioural relationships.

We assume that there may be two types of interaction: the “investment’s interaction” and the “financial intermediaries’ interaction”.

1. During the investment’s interaction, the investment share may influence real GDP per capita. The financial intermediaries, in presence of asymmetric information, may affect the investment share, and, in turn, produce an improvement in the economic performance. Putting this “investment’s interaction” into equation format, we have:

\[
LY = \beta_{13} LINV + \beta_{14} Trend \quad (3)
\]

\[
LINV = \beta_{22} LPC + \beta_{24} LINF + \beta_{25} Trend \quad (4)
\]

In other words, we are assuming that, considering our four variables, there may be two cointegrating vectors between our series. We expect that the first cointegrating vector shows a positive relationship between real GDP per capita and the investment share (or a positive relationship between real GDP per capita and the investment share expressed in real terms, in the case in which we also consider the inflation in the first vector) and a positive association between real GDP per capita and trend (where trend is interpreted as technical progress). Furthermore, we expect that the second cointegrating vector shows a positive relationship between the investment share and the financial development indicator and a negative associative between the investment share and the inflation.

2. The “financial intermediaries’ interaction” is based, instead, on the transmission of an information probably made distort by the presence of inflation. In other words, the information which financial intermediaries possess when deciding on the choice of investment may be altered by the presence of the inflation. In this case, the inflation may interfere negatively with resource allocations made by financial intermediaries and, in turn, may produce a negative economic performance.
To sum up this last interaction, we expect to detect two cointegrating vectors with one vector being economic growth equation and the other one being the relation between financial development indicator and inflation. Also in this second interaction, the trend is interpreted as technical progress.

\[
LY = \beta_{12}LPC + \beta_{15}Trend
\]  
(5)

\[
LPC = \beta_{23}LINV + \beta_{24}LINF + \beta_{25}Trend
\]  
(6)

Concerning the econometric issue of cointegration, it is useful to start our discussion by defining the concept of integration.

For a scalar \( x_t \), the first order autoregression implies

\[
x_t = \pi_t x_{t-1} + \varepsilon_t
\]  
(7)

This can be rewritten as

\[
\Delta x_t = \pi x_{t-1} + \varepsilon_t
\]  
(8)

where \( \pi = \pi_1 - 1 \) by subtracting \( x_{t-1} \) from both sides of (7). If \( \pi_1 = 1 \) or equivalently \( \pi = 0 \), then \( x_t \) has a unit root and is said to be differenced once to achieve stationarity. In the case simple of (7), \( x_t \) is a random walk if it has a unit root. If \( |\pi_1| < 1 \), then \( x_t \) is stationary.

For general autoregressive processes, (7) includes additional lags of \( x_t \); thus (8) includes lags of \( \Delta x_t \). Equation (7) can be generalized to represent a vector of variables and to include higher-order lags of \( x_t \). Together, these result in:

\[
x_t = \sum_{i=1}^{l} \pi_i x_{t-i} + \varepsilon_t
\]  
(9)

where \( \varepsilon_t \sim IN(0, \Omega) \), and \( l \) is the maximum lag, and (9) may include a constant and dummies as well.

Following Johansen (1988) and Johansen and Juselius (1990), (9) provides the basis for cointegration analysis. By adding and subtracting various lags of \( x_t \), (9) can be rewritten as:

\[
\Delta x_t = \pi x_{t-1} + \sum_{i=1}^{l} \Gamma_i \Delta x_{t-i} + \varepsilon_t
\]  
(10)
where the \( \{ \Gamma_i \} \) are \( \Gamma_i = -(\pi_{i1} + \ldots + \pi_{il}) \), \( i = 1, \ldots, l - 1 \) and

\[
\pi = \left( \sum_{i=1}^{l} \pi_{ii} \right) - I \tag{11}
\]

As in (8), \( \pi \) in (10) could be zero. If so, \( \Delta x \) in (10) depends upon \( \varepsilon \), and lags of \( \Delta x \), alone, all of which are I(0); so \( x \) is I(1). If \( \pi \) is not zero and of full rank with all the roots of an associated polynomial being within the unit circle, then all the \( x \) are I(0), paralleling \( |\pi_i| < 1 \) in the univariate case. However, because \( \pi \) is a matrix in (10) rather than a scalar, \( \pi \) may be of less than full rank, but of rank greater than zero. If so, each of the variables in \( x \) can be I(1), even while some linear combinations of those variables are I(0). The variables in \( x \) are then said to be cointegrated.

To show how cointegration can occur, denote the dimension of \( x \), as \( p \times 1 \) and the polynomial \( \left( \sum_{i=1}^{l} \pi_i z^i \right) - I_p \) as \( \pi (z) \), where \( z \) is the argument of the polynomial. Note that \( \pi = \pi (1) \), from (11). The three possibilities for rank \( (\pi) \) are as follows.

1. \( \text{rank}(\pi) = p \)

For \( \pi \) to have full rank, none of the roots of \( |\pi (z^{-1})| = 0 \) can be unity. Provided \( |\pi (z^{-1})| = 0 \) has all its \( l \cdot p \) roots strictly inside the unit circle, \( x \) is stationary because \( \pi (L) \) can be inverted to give an infinite moving average representation of \( x \).

2. \( \text{rank}(\pi) = 0 \)

Because \( \pi = 0 \), equation (8) is in differences only, and there are \( p \) unit roots in \( |\pi (z^{-1})| = 0 \).

3. \( 0 < \text{rank}(\pi) \equiv r < p \)

In this case, \( \pi \) can be expressed as the outer product of two (full column rank) \( p \times r \) matrices \( \alpha \) and \( \beta \):

\[
\pi = \alpha \beta' \tag{12}
\]

and there are \( p - r \) unit roots in \( |\pi (z^{-1})| = 0 \).
In (12), $\beta$ is the matrix of cointegrating vectors, and $\alpha$ is the matrix of “weighting elements”. Substituting $\pi = \alpha \beta' \pi$ into $\Delta x_t = \pi x_{t-1} + \sum_{i=1}^{r-1} \Gamma_i \Delta x_{t-i} + \varepsilon_t$ gives

$$\Delta x_t = \alpha \beta' x_{t-1} + \sum_{i=1}^{r-1} \Gamma_i \Delta x_{t-i} + \varepsilon_t$$  \hspace{0.5cm} (13)

Each $1 \times p$ row $\beta_i'$ in $\beta'$ is an individual cointegrating vector, as is required for “balance” to make each cointegrating vector $\beta_i' x_{t-1}$ an I (0) process in (13). Each $1 \times r$ row $\alpha_j$ of $\alpha$ is the set of weights for the $r$ cointegrating terms appearing in the $j^{th}$ equation.

Thus, the rank $r$ is also the number of cointegrating vectors in the system. While $\alpha$ and $\beta$ themselves are not unique, $\beta$ uniquely defines the cointegration space and suitable normalisations for $\alpha$ and $\beta$ are available.

In essence, $\alpha \beta' x_{t-1}$ in (13) contains all the long-run (level) information on the process for $x_t$. The vector $\beta_i' x_{t-1}$ measures the extent to which actual data deviate from the long-run relationship among the variables in $x_{t-1}$.

In order to determine the value of $r$, Johansen and Juselius developed a maximum likelihood-based testing procedure and tabulated the asymptotic critical values of the likelihood ratio (LR) statistic as a function of $p-r$. This statistic generalises the Dickey Fuller statistic to the multivariate context. Two variants of the LR statistic exist, one using the maximal eigenvalues over a subset of smallest eigenvalues (the maximal eigenvalues statistic), the other using all eigenvalues in that subset (the trace statistic).
4. Empirical Evidence: the case of Japan

This section attempts an evaluation of the growth models illustrated in section 2. We shall start our analysis with a brief history of the Japanese economic development.

Forty years ago, Japan was not one of the richest countries of the world: its per capita GDP was lower than the US and the EU one. Japanese economic performance started to be positive from the ‘60s. The Japanese economic success has to be attributed to its own internal conditions and its adoption of outward-oriented development strategy. In the first half of the ‘50s, in fact, the government and the central bank adopted important policies devoted both to the promotion of technology import and to favourable tax treatment for capital investment. The high economic growth experienced by Japan in the ‘60s was hence supported by rapid expansion of capital investment in the private sector, by abundant supply of high quality labour enabled by a large transfer of the working population from the primary industry to the secondary industry and by import of foreign technology. The fourth Middle East War in October 1973 brought, however, the first oil crises and, as a result, Japan’s economy recorded negative growth in 1974 for the first time in the post war period. In the second oil crises (1979), efforts were made to change its industrial structure. In 1980, initiatives in administrative reform and privatization had the effect of revitalizing the private corporate sector. At the beginning of 1990, a new recession occurred due to the internationalisation of financial and stock markets and the collapse of the bubble economy.

4.1 The data set

In this section, we present the methodology and the result for Japan. The data, obtained from the online information system ESDS International - International Financial Statistics (IMF, 2004), is in quarterly format and spans a period of 46 years from 1957 to 2003.

4.1.1 Testing for Nonstationarity

The preliminary step of our analysis is to check whether the time series variables are non stationary. For this purpose, a test of the null hypothesis of nonstationarity is conducted via the well-known Dickey-Fuller procedure. We run the augmented Dickey-Fuller (ADF) tests in the level and first differences, with trend, constant and seasonal
dummies. The null hypothesis is that the variables in question contain unit root and the alternative hypothesis is that the variables are trend stationary.

The ADF statistics suggests that all variables are I (1). The hypothesis of a unit root in real GDP per capita, in financial measure, in investment share and in inflation cannot be rejected for Japan. Hence, all the variables are non-stationary in level form. The variables are therefore differenced and the ADF run again. The null hypothesis of unit roots in the first differences can be rejected. The ADF tests in table 1 show that the variables in the level are stationary. ADF is the augmented Dickey-Fuller statistic, k is the degree of augmentation determined following the general-to-specific procedure suggested by Campbell and Perron (1991).

<table>
<thead>
<tr>
<th>Table 1. Unit root tests for LY, LPC, LINV and LINF</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Country</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Japan</td>
</tr>
<tr>
<td>k</td>
</tr>
</tbody>
</table>

4.2 Cointegration in the VAR

To find which variables adjust to the long run cointegrating relations, we focus on cointegration in the Vector Autoregressive model (VAR). The VAR model will provide a feasible empirical system for the analysis of our integrated economic time series.

As a first step in the analysis, an unrestricted VAR (8) model, with constant term, trend and seasonal dummies, is estimated by OLS. In order to check the adequacy of the VAR, we look at mis-specification tests.

The table 2 reports the output about tests of mis-specification for the variables real GDP per capita (LY), private Credit (LPC), investment share (LINV) and inflation (LINF).

5 We began by choosing a VAR of order 10 and, after performing the general to specific procedure, we arrived at the estimation of VAR of order 8.
Table 2. Mis-specification Tests

<table>
<thead>
<tr>
<th></th>
<th>LY</th>
<th>LPC</th>
<th>LINV</th>
<th>LINF</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR 1-5 test</td>
<td>0.47709[0.7928]</td>
<td>0.89335[0.4877]</td>
<td>1.0893[0.3695]</td>
<td>1.1912[0.3171]</td>
</tr>
<tr>
<td>Normality test</td>
<td>72.665[0.0000]**</td>
<td>134.45[0.0000]**</td>
<td>5.7187[0.0573]</td>
<td>41.930[0.0000]**</td>
</tr>
<tr>
<td>ARCH 1-4 test</td>
<td>0.076109[0.9894]</td>
<td>0.068257[0.9914]</td>
<td>0.24676[0.9112]</td>
<td>0.34982[0.8437]</td>
</tr>
<tr>
<td>Hetero test</td>
<td>0.64371[0.9625]</td>
<td>0.98846[0.5186]</td>
<td>0.75560[0.8719]</td>
<td>0.87134[0.7117]</td>
</tr>
</tbody>
</table>

The AR (1- m) is a test of residual autocorrelation of order m distributed as $F(m,T)$, i.e. a test of $H_0: u_t = \epsilon_t$ against $H_1: u_t = \rho_1u_{t-1} + ... + \rho_mu_{t-m} + \epsilon_t$. The test of autocorrelated errors of order 1-5 for all the variables under consideration is small and the null of no autocorrelation is clearly accepted. Normality denotes the test of residual normality, distributed as $\chi^2(2)$. It is based on the third and the fourth moments around the mean, i.e., it tests for skewness and excess kurtosis of the residuals. The normality of the residuals is rejected for the variables LY, LPC and LINF.

The ARCH (m) is a test of autoregressive residual heteroscedasticity of order m distributed as $F(m,T-m)$. As we can see, there is no presence of ARCH effect in our data.

The heteroscedasticity test involves an auxiliary regression of the squared residuals on the original regressors and all their squares. The null is unconditional homoscedasticity, and the alternative is that the variance of the error process depends on the regressors and their squares. The null hypothesis of homoscedasticity is accepted by the data.

To sum up, the standard assumptions underlying the VAR are satisfied, expect for the normality tests. In Figure 2, we have graphed the residuals and the residual histogram compared with the normal distribution.
By a graphical inspection of the residuals, the rejection of normality seems to be due to the presence of several outliers in the data. This is further confirmed by the histograms exhibiting long tails. The largest outlier observation is recorded at 1974(1), the quarter after the Middle East War (October 1973). To take into account for this abnormal observation, we introduce a dummy variable, $D_{74:1}$, defined as unrestricted.

We also detect four additional “outlier” observations accounted for by dummy variables defined as follows: the dummy variable $D_{68:4}$, which takes value one for 1968 (4) and zero otherwise; the dummy variable $D_{63:1}$, which takes value one for 1963 (1) and zero otherwise; the dummy variables $D_{98:4}$ and $D_{60:2}$.

The estimates of the URF residuals show a high correlation between the actual and fitted values (0.99977, 0.99958, 0.98792, and 0.99995) and the residual standard errors are very low (0.011042, 0.013193, 0.015753, 0.0070509). Thus the VAR fits well. Having taking into account the “outlier” observations, the normality is not longer a
problem and it is possible to see that the distribution of the residuals become much closer to a normal distribution than in the first tentatively estimated model.

Table 3. Mis-specification Tests with the intervention of dummies variables

<table>
<thead>
<tr>
<th></th>
<th>LY</th>
<th>LPC</th>
<th>LINV</th>
<th>LINF</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR 1-5 test</td>
<td>2.1531[0.0635]</td>
<td>2.3841[0.0420]*</td>
<td>0.83485[0.5273]</td>
<td>0.52448[0.7574]</td>
</tr>
<tr>
<td>Normality test</td>
<td>0.98717[0.6104]</td>
<td>3.2531 [0.1966]</td>
<td>5.2197 [0.0735]</td>
<td>3.8886 [0.1431]</td>
</tr>
<tr>
<td>ARCH 1-4 test</td>
<td>1.5682 [0.1871]</td>
<td>1.1122 [0.3540]</td>
<td>0.19228[0.9420]</td>
<td>1.5622 [0.1887]</td>
</tr>
<tr>
<td>hetero test</td>
<td>0.60085[0.9786]</td>
<td>0.82804[0.7747]</td>
<td>0.60965[0.9755]</td>
<td>0.64324[0.9605]</td>
</tr>
</tbody>
</table>

However, the empirical densities still show longer tails than the normal density and the empirical model for the variable private credit still show some evidence of autocorrelation. We can note that the autocorrelation coefficients for the variable private credit are not very small (figure 4). That means that the truncation to the lag 8 is not very appropriate for this variable. The estimation with recursive least squares does not facilitate the stability analysis. However, we decide to proceed our analysis.

Figure 3. Residuals scaled and Residual histograms
Given the results obtained so far, we decide to continue our analysis by testing for cointegration in unrestricted system. As Hendry and Juselius (2000) noted, the correct choice of the cointegration rank is far from being easy. The information that we need to consider when deciding on the choice of cointegration rank are the following:

- the trace test for cointegration rank; the null hypothesis is that there is no cointegrating vector and the alternative is that there is one cointegrating vector;
- the recursive graph; the recursively estimated components of the trace statistic should increase linearly for the first \( r \) components, but stay constant for the remainder;
- the graph of the cointegrated relations: the graphs should look stationary, otherwise we should reconsider the choice of the rank;

In the following discussion, we consider all this set of information. But before determining the rank, another important issue has to be considered: the issue of the status of the intercept and of the trend. In this context, Doornik and Hendry suggest to test the following hypotheses:

\[
\begin{array}{c|cccc}
\text{Table 4. Set of hypotheses} \\
p=0 & p \leq 1 & p \leq 2 & p \leq 3 \\
H_{q1}(p) & 67.951 [0.002] ** & 37.196 [0.028] * & 9.6253 [0.525] & 0.0205 [0.886] \\
H_{l}(p) & 98.648 [0.000] ** & 44.381 [0.034] * & 15.435 [0.546] & 0.57340 [0.999] \\
\end{array}
\]
The first row represents the case in which trend and intercept are unrestricted in the VAR model. This hypothesis corresponds to the model with linear trends in the first difference and a quadratic trend in the level. The second row of table 4 represents the hypothesis in which trend is restricted to lie in the cointegration space, but constant is unrestricted in the model. The implication of this hypothesis is the presence of linear deterministic trend in the level of our variables. Looking at the table 4, we accept $H_0(2)$. The second step, most crucial, is test for $r$ cointegrating vector. The test used here is based on the maximum likelihood approach and discriminate between two hypotheses: full rank or cointegration relations. The trace test reports $r=2$. As mentioned, the acceptance of this rank requires many analyses.

By the inspection of the graphs in the figure 5, which show the two cointegrating vectors, we can notice that the first one looks more stationary than the second one. Furthermore, the first cointegration relation fits better than the second one. The recursive graphs in figure 6 show that the second estimated eigenvalues remains constant, whereas the first one is relatively constant.

*Figure 5. Graphs of the unrestricted cointegration relations $\beta_1'y_i$ and $\beta_2'y_i$*
Looking at the limit distribution under the null hypothesis of cointegrating rank which depends on nuisance parameters (the presence of the trend and the value of the actual cointegrating rank), there should be also two cointegration relations.

<table>
<thead>
<tr>
<th>$r$</th>
<th>$\text{eigenvalues}$</th>
<th>Trace statistic</th>
<th>95% quantile</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.27192</td>
<td>98.648</td>
<td>47.21</td>
</tr>
<tr>
<td>1</td>
<td>0.15573</td>
<td>44.381</td>
<td>29.38</td>
</tr>
<tr>
<td>2</td>
<td>0.083238</td>
<td>15.435</td>
<td>15.34</td>
</tr>
<tr>
<td>3</td>
<td>0.0033476</td>
<td>0.57340</td>
<td>3.89</td>
</tr>
</tbody>
</table>

The two trace statistics are larger than their 95% quantiles. This implies that both series should be considered stationary. We remind that the eigenvalues measure the size of the coefficients of the cointegrating relationships, and they can be interpreted as the squared canonical correlation coefficients. In this sense, the magnitude of the eigenvalues is an indication of how strongly the linear combination $\beta'y$ is correlated with the stationary part of the process. As it appears from the table 5, both of eigenvalues are quite small; in particular, the second one is smaller than the first one. In this situation, we check the economic interpretability of the second cointegrating relation, in order to see if it contains valuable information for the analysis.
Under this perspective, we can make interesting observations from the values of adjustment coefficients. In fact, imposing the following restriction $\alpha_{21} = \alpha_{23} = 0$, which is accepted by the data ($\chi^2(2) = 5.4632 [0.0651]$), it emerges that the measure of financial development would influence real GDP or the investment share, but it would not be influenced by them. Furthermore, considering that $\alpha_{11} = 0.05$ and $\alpha_{31} = 0.018$, it seems evident that real GDP adjusts more quickly to the changes in the investment share and that the investment share adjusts more quickly to the changes in the private credit. This would imply that the private credit measure influenced the investment share, which influenced real GDP per capita$^6$.

From these considerations, we decide to continue our analysis assuming $r = 2$.

Given the choice of the number of cointegrating relations, we start the simplification of our model, imposing restrictions on the long run parameters $\beta$ and on the short-run adjustment parameters $\alpha$. Therefore, the central part of our long run cointegration analysis is to impose restrictions on $\beta$ to achieve economic interpretability.

The hypotheses on the cointegration vectors are formulated in spirit of the models developed in the paragraphs 2.1, 2.2 and 3.2.

According to the models developed in the Section 2.1 and 3.2, we focused on the investment share as key link between financial development and economic growth and, specifically, we examine whether the financial development may affect real GDP per capita through the investment share.

The exogeneity tests allow us to impose weak exogeneity of the financial development measure to the system ($\chi^2(2) = 5.4632 [0.0651]$). The next step is to impose the following normalizations $\beta_{11} = 1, \beta_{23} = 1$: the first vector is normalised on real GDP per capita and the second vector is normalised on the investment share. Carrying on the identification of our system, we drop the logarithm of financial development from the first vector ($\beta_{12} = 0$), the restricted form of which can be interpreted as an economic growth equation. Concerning the second vector, we drop real GDP per capita, but we are not able to drop the trend. The final restriction, which is acceptable by the

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$^6$ As Enders (1995, pg.359) noted, cointegration existence should not be interpreted as an ex ante
data, is that real GDP is influenced by real investment share ($\beta_{13} = -\beta_{14}$). Turning to the economic interpretation of these results, which are not very straightforward, it may first be noted that the financial development has a positive impact on the volume of the investment, independently of the inflation which has a positive effect on it (see equation 2). The first cointegrating vector provides support for a positive impact of real investment on real GDP (see equation 14).

\[
LY = 0.17(LINV - LINF) - 0.013Trend \\
LINV = 0.29LPC + 0.19LINF - 0.0012Trend
\]

What is rather surprising in these findings is the positive relationship between the volume of investment and the level of inflation (in the equation 15). This contradicts the macroeconomics thesis which predicts a negative association between the investment and inflation.

According to the theoretical model developed in the Section 2.2 and 3.2, which focuses on the negative impact of inflation on the financial development, which, in turn, traduce in low economic growth, we impose the following restrictions to achieve the economic interpretability of our cointegrating relationships.

By looking at the adjustment coefficients, we are inclined to say that the inflation affected the financial development which influenced real GDP per capita negatively.

Turning to restrictions suggested by the theory, we normalised the first vector on real GDP per capita and the second vector on the financial development indicator. We would expect that the first vector explains real GDP in terms of financial development and the second vector appears to be an equation which explains long run financial development in terms of inflation. As we can note, the results of our imposed restrictions seems to be consistent with the prediction of the theoretical models:

\[
LY = -0.38LPC + 1.97LINV - 0.012Trend \\
LPC = 1.93LINV - 0.75LINF + 0.0075Trend
\]

The second cointegrating vector shows a negative association between financial development measure and the level of inflation.
4.3 Results’ interpretation

Two links have been particularly emphasized in our analysis: the relationship between real GDP per capita and the investment share and the relationship between real GDP per capita and financial development performance.

The statistical analysis described in the body of this section shows that real GDP responds positively to the share of real investment (see equation 14) and the investment share responds positively to the financial development performance (see equation 15). On the contrary, the financial system responds negatively to an increase in the inflation (see equation 17), and real GDP responds negatively to LPC (see equation 16).

These findings might also suggest that real GDP is affected mainly by the share of the investment rather than by the financial development (in the equation 16).

A further complication to the economic interpretation of our findings arises from the rejection of the weak exogeneity of the investment share and the level of inflation. In view of the weakness of the results, we would suggest that there is a great deal of work to do in this area. Such work is likely to provide more fertile results including variables able to take into account the stock market system. In this context, we would suggest, as other possible variables, to include in the present study, the volatility of inflation and the stock market capitalisation, to account for the peculiar role played by the stock market in Japan.

An improvement of this study could be also obtained using different measures of financial development and a different measure of investment. For example, we would suggest using investment efficiency rather than investment share. One way to measure the efficiency of investment is to calculate the change in per capita output divided by the change in domestic capital stock\(^7\). In order to minimize omitted variables bias and to check the robustness of the results, we would also suggest including measures of monetary policy, fiscal policy and trade policy.

Finally, we would suggest that the empirical links between financial development and economic growth deserve further investigation. One way to improve the analysis may be to re-estimate the model with full efficiency. Under this

\(^7\) This measure is suggested by Tirivavi, (2000).
perspective, it is advisable to perform an unrestricted VAR in (stationary) differences of our variables, adding the obtained cointegration relationships.

5. **Empirical evidence: the case of the US**

If we looked at the US average rate of growth per capita over the period 1957-2003, we would notice that it has reduced from the mid-1970s until the mid-1990s and it has started to increase slowly from 1995.

The US growth path may lead one to ask why this slowdown in the growth has occurred. For several economists, the reasons do not seem to be in specific characteristics of the US economy, since most of the developed countries experienced similar patterns. They support the view that rich countries are losing their economic power and poor countries grow faster than rich ones. Others support that this phenomenon is due mainly to scarce investments and to insufficient capital accumulation.

5.1 **The data set**

In this section, along the line of the analysis developed in section 4, we present the methodology and the result for the US. The data, as in the Japanese case, is obtained from the online information system ESDS International -International Financial Statistics (IMF, 2004). The data consist of four variables: the measure of financial development is private credit. The inflation measure is denoted by LINF and it is given by the Consumer price. The investment is measured by the ratio of gross capital formation to nominal GDP, and the output is represented by real GDP per capita. The data is quarterly from 1957 (2) to 2003 (4). This gives us a total of 187 observations. The data are transformed logarithmically. Cointegration technique is applied to evaluate the long-run hypotheses described in the section 3.2. The estimation is carried out by PcGive.

5.1.1 **Testing unit root and cointegration**

Before modelling the data, we consider its basic stationary properties. All series appear to be I (1). The test of the null hypothesis of nonstationarity is conducted via Dickey-Fuller procedure. As in the Japanese case, we run the augmented Dickey-Fuller
(ADF) tests in the level and first differences, with trend, constant and seasonal
dummies. The null hypothesis that the variables in question contain unit root cannot be
rejected. Therefore, all the variables are non-stationary in level form. Differencing our
variables and running ADF again, the null hypothesis of unit roots in the first
differences can be rejected. The ADF test in table 6 shows that the variables in the level
are stationary. The general-to-specific procedure suggested by Campbell and Perron
(1991) has been used to determine the number of lags.

<table>
<thead>
<tr>
<th>Table 6. Unit root tests for LY, LPC, LINV and LINF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>ADF</td>
</tr>
<tr>
<td>k</td>
</tr>
</tbody>
</table>

The next step is to establish the cointegration properties of the system. To
achieve this, we start out with the vector autoregression approach of Johansen (1988)
and Johansen and Juselius (1990). An unrestricted VAR (10) with constant term, trend
and seasonal dummies is the starting point of the “general-to-specific” search that is
accepted by the progress output of PcGive. The strive for parsimony of the model has
resulted in the inclusion of seasonal dummies, since they are necessary. Again, to
understand if the VAR is an adequate statistical representation of the data, we look at
the mis-specification tests. Unlike the Japanese case, these mis-specification tests
clearly reveal that the VAR chosen is statistically adequate. Normality is not rejected.
The test for autocorrelation shows absence of autocorrelation. Furthermore, there is the
presence of homoscedasticity and the absence of ARCH effect.

<table>
<thead>
<tr>
<th>Table 7. Mis-specification Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>AR 1-5 test</td>
</tr>
<tr>
<td>Normality test</td>
</tr>
<tr>
<td>ARCH 1-4 test</td>
</tr>
<tr>
<td>hetero test</td>
</tr>
</tbody>
</table>
The graphical inspection of the residuals confirms these results. Hence, the assumptions (normally distributed errors, non serially correlated residuals and residual homoscedasticity) on which the model is based, are satisfied.

Nevertheless, the determination of the cointegration rank is made difficult by the many hypotheses that can be formulated and by the non standard limit distribution. The limit distribution of the trace statistic depends on the presence or absence of the trend.

In order to establish if our model contains a linear deterministic trend, we test two hypotheses, with trend and intercept unrestricted and with restricted trend and unrestricted constant. Like in the Japanese case, we accept the model with trend restricted to lie in the cointegration relation and with unrestricted constant.

Figure 8. Residuals (scaled) and residual histograms

Table 8. Set of hypotheses

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>$p=0$</th>
<th>$p\leq 1$</th>
<th>$p\leq 2$</th>
<th>$p\leq 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1(p)$</td>
<td>77.829 [0.000] **</td>
<td>36.961 [0.030] *</td>
<td>9.5625 [0.531]</td>
<td>2.8228 [0.093]</td>
</tr>
<tr>
<td>$H_0(p)$</td>
<td>88.381 [0.000] **</td>
<td>44.437 [0.033] *</td>
<td>14.530 [0.620]</td>
<td>6.4889 [0.411]</td>
</tr>
</tbody>
</table>
To make sure that the size of the test based on the trace statistic has the correct value for all parameter points in $H_I$, that the trend is present, we apply the quantiles.

Table 9. The eigenvalues trace statistics and 95% quantiles for the US data.

<table>
<thead>
<tr>
<th>$r$</th>
<th>Eigenvalues</th>
<th>Trace statistic</th>
<th>95% Quantile $^8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.22897</td>
<td>88.381</td>
<td>47.21</td>
</tr>
<tr>
<td>1</td>
<td>0.16219</td>
<td>44.437</td>
<td>29.38</td>
</tr>
<tr>
<td>2</td>
<td>0.046465</td>
<td>14.530</td>
<td>15.34</td>
</tr>
<tr>
<td>3</td>
<td>0.037668</td>
<td>64.889</td>
<td>3.89</td>
</tr>
</tbody>
</table>

The idea is to use not one test statistic to reject $H_I$, but two, namely the trace statistic compared to its quantile. This procedure guarantees that the asymptotic size of the test is correct. In our case, we conclude, even if on the basis of the output progress, that the trend is needed to describe the data. The cointegration rank can be formally estimated as the smallest $r$ that is not rejected at a given level of significance. In the present example (see table 9), we can reject $r = 0$, since the test statistic is 88.381 and the quantile is only 47.21. We can also reject $r = 1$ because the trace statistic is greater than the corresponding quintile in the asymptotic distribution. The hypothesis $r = 2$ can be accepted. Hence, the trace statistics suggests the presence of two cointegrating relationships: the estimate of $\alpha$ and $\beta$ are given as the first two columns of the adjustment coefficients and of the eigenvectors, respectively. The two graphs below show the two cointegrating vectors. Unlike the Japanese case, they look more stationary and they fit better.

$^8$ The table of the quantiles of the likelihood ratio test for unrestricted constant is obtained from Johansen, (1995).
Therefore, according to the trace statistics and the graphical analysis, we can conclude that $r = 2$ can be accepted by the data.
In order to understand which the driving forces that cause an improved
economic performance are, we look at the economic interpretability of the adjustment
coefficients and at the values of cointegrating vectors. Imposing the restrictions
suggested by the models developed in Section 2, the analysis reveals contradictory
results.

Given those models, real GDP per capita is hypothesized to be cointegrated with
the investment share or/and with the financial development indicator. Furthermore, we
assume that the investment share and the financial development indicator is cointegrated
with the financial system and with inflation, respectively. The weak exogeneity of the
regressors is required for our analysis to be efficient. This hypothesis is formulated as a
parametric restriction on the adjustment coefficients. We calculate the statistic to test
the weak exogeneity of each variable, LY, LPC, LINV and LINF, in the hope that we
can justify the analysis of the equation system. It is seen, unfortunately, that real GDP
per capita is weakly exogenous (Chi^2(2) = 4.2900 [0.1171]), and that none of the three
remaining variables appear to be weakly exogenous to the system. The lack of
exogeneity of the variables LPC, LINV and LINF would not allow investigating our
model. But as simple illustration of the cointegration technique, we assume that the
three variables are weakly exogenous.

According to the first model developed in 2.1 and 3.2, which identifies in the
investment share the channel that links the financial development to the economic
growth, the first eigenvector is normalized with respect to LY, which looks like to the
economic growth equation. The second eigenvector is normalized with respect to the
investment share. Then, the identification of our model requires that we drop the
logarithm of private credit measure from the first vector, and real GDP from the second
vector. The attempt to drop the trend from the second vector is rejected. Furthermore, a
simple and reasonable restriction is imposed: we assume that it is the investment share
in terms real to affect real GDP (β_{13} = -β_{14}).
Table 10. Cointegration: normalized $\alpha$ and $\beta'$ matrixes

<table>
<thead>
<tr>
<th>Hypotheses</th>
<th>Eigenvectors $r=0$</th>
<th>Eigenvectors $r \leq 1$</th>
<th>Adjustment coefficients $r=0$</th>
<th>Adjustment coefficients $r \leq 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LY</td>
<td>1.00</td>
<td>6.64</td>
<td>0.042</td>
<td>0.003</td>
</tr>
<tr>
<td>LPC</td>
<td>-1.18</td>
<td>1.28</td>
<td>0.07</td>
<td>-0.025</td>
</tr>
<tr>
<td>LINV</td>
<td>-0.022</td>
<td>1.00</td>
<td>0.01</td>
<td>-0.04</td>
</tr>
<tr>
<td>LINF</td>
<td>-1.69</td>
<td>3.02</td>
<td>0.008</td>
<td>0.003</td>
</tr>
<tr>
<td>Trend</td>
<td>0.006</td>
<td>-0.05</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The restricted cointegrating model takes the following form:

$$LY = 0.18(LINV - LINF) + 0.007Trend$$  \hspace{1cm} (18)

$$LINV = -3.07LPC - 4.7LINF + 0.03Trend$$  \hspace{1cm} (19)

The economic interpretation is rather more complex. According to the first cointegrating vector, real GDP is positively related to the real investment share and positively related to trend. The second vector shows a negative relationship between the investment share and private credit and also a negative relationship between the investment share and inflation (as the economic theory predicts).

If LPC, LINV and LINF were weakly exogenous, we would be inclined to say that the real GDP is determined by the investment share in the long run, and that there exist a negative association between the investment share and the level of inflation.

According to the model developed in Section 2.2, which stresses the importance of the effect of inflation on the financial development performance, we repeat the application of reduced rank regression estimating all the parameters. From economic reasoning, it seems plausible to normalize the first vector on real GDP per capita and the second vector on the financial development indicator.

Table 11. Cointegration: normalized $\alpha$ and $\beta'$ matrixes

<table>
<thead>
<tr>
<th>Hypotheses</th>
<th>Eigenvectors $r=0$</th>
<th>Eigenvectors $r \leq 1$</th>
<th>Adjustment coefficients $r=0$</th>
<th>Adjustment coefficients $r \leq 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LY</td>
<td>1.00</td>
<td>5.18</td>
<td>0.02</td>
<td>0.004</td>
</tr>
<tr>
<td>LPC</td>
<td>-1.18</td>
<td>1.00</td>
<td>0.07</td>
<td>-0.032</td>
</tr>
<tr>
<td>LINV</td>
<td>-0.022</td>
<td>0.78</td>
<td>0.09</td>
<td>-0.05</td>
</tr>
<tr>
<td>LINF</td>
<td>-1.69</td>
<td>2.36</td>
<td>0.008</td>
<td>0.004</td>
</tr>
<tr>
<td>Trend</td>
<td>0.006</td>
<td>-0.04</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In order to obtain the identification of our model, we drop the logarithm of inflation from the first vector and the logarithm of real GDP per capita from the second vector.

\[
LY = 0.09LPC - 0.11LINV + 0.005Trend
\]

(20)

\[
LPC = -0.12LINV - 1.56LINF + 0.011Trend
\]

(21)

The findings are not consistent with the prediction of the economic theory.

The first cointegrating vector shows a positive relationship between real GDP and financial development indicator and a negative relationship between real GDP per capita and the investment share. The second cointegrating vector shows that the financial system is related negatively to the investment share and to the inflation rate.

As already mentioned, this economic interpretation has to be regarded with distrust, since there is the lack of the weak exogeneity of our regressors. Interestingly, real GDP, which should be not exogenous to the system, appears to be weakly exogenous with respect to each of our variables. If the cointegration analysis could say anything about causality, we could say that in the United States, real GDP causes financial development.

5.2 Results' interpretation

Unlike the Japanese case, the presence of real GDP as weakly exogenous variable makes the analysis more difficult, not because the method is more complicated but because it drives as away from our theoretical models.

Anyway, it is important to note that, if we consider the weak exogeneity of real GDP, the investment share would be driven by financial development indicator and by real GDP. Although the cointegration analysis does not say anything about causality, we would maintain that, in Japan, the financial development affected real GDP per capita, whereas, in the US, it seems that real GDP per capita affected the financial development. We could arrive at this conclusion, considering that in the Japanese case, private credit is resulted to be weakly exogenous, while, in the US case, real GDP is weakly exogenous.

Furthermore, considering that most researchers would consider this outcome quite unsatisfactoring, we would like to underline that, as far as this preliminary paper is concerned, the US case study should be considered solely as an illustration of the
cointegration technique potentials, where the details have to be worked out yet and it will take time to explore the data and the methods which can be applied with success. The same considerations, made at the end of section 4, about different measures of financial development and about potentially excluded variables in the analysis can be also applied to the US case.

6. Some concluding remarks

In this work we have reviewed empirical evidence on the relationship between financial development and economic growth. This study has modelled real GDP with financial and macroeconomic variables in two countries: the US and Japan.

Cointegration technique, proposed by Engle and Granger (1987) and extended by Johansen (1988), has been applied to evaluate the long-run hypothesis that our variables are cointegrated. The basic idea is that individual time series wander considerably but economic forces tend to make these series stationary. Given the basic economic model, real GDP per capita has been hypothesized to be cointegrated with the investment share, inflation and financial development indicator.

Two economic models were tested and they have satisfied a range of statistical criteria. The cointegrating properties of our time series have been analysed from the reduced form of the model and we tested hypotheses about the coefficients of the cointegrating relations. Finally, we tested the economic questions and hypotheses against the data.

Illustrating our concerns about the relationship between financial development and growth, we offered empirical evidence in the US and Japan, which revealed important differences in the links between finance and growth.

The Japanese case appeared to be a satisfactory representation of the relationship between finance and growth. In large part, real GDP seemed to be determined by the investment share in the long run. The selected model in the US case did not have a suitable economic interpretation, although it satisfied all statistical analysis.

There are many other questions that we should take into consideration in further development of this study. However, it is worth mentioning that the examination of the impact of financial development and investment on growth in the cointegration framework seems to offer new suggestions for future research.
REFERENCES


